

## Generalizing Truth-Functionality

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A common standard for the interpretation of classical propositional logic is set by the functionally complete collection of 2-valued truth-tables. The structure of the free algebra of formulas is faithfully mirrored in the semantics: In each admissible model, each sentential letter freely ranges over the set of truth-values, and to each  $n$ -ary logical constant there corresponds a convenient  $n$ -ary operator over those same truth-values. The whole approach is easily generalizable so as to define the class of *truth-functional logics*, i.e., *many-valued* logics whose operators can be characterized by *truth-tables* over some convenient set of truth-values.

A further interesting generalization of the above idea is produced by the so-called *non-deterministic truth-tabular semantics* (*N-truth-tables*), where each  $n$ -ary operator is allowed to choose in between a number of possible outputs for each given  $n$ -uple of inputs. The set of admissible models may thus be enlarged, allowing for the natural adequate interpretation of more generous classes of non-classical logics. Yet another generalization of the same idea is given by the so-called *possible-translations semantics with many-valued ingredients* (Many-valued PTS), where each model of a logic  $\mathcal{L}$  is given by an admissible translation of  $\mathcal{L}$  into an appropriate many-valued logic  $\mathcal{L}_k$  coupled with a standard many-valued valuation from  $\mathcal{L}_k$ .

In the finite-valued case, logics with truth-tabular or N-truth-tabular semantics share interesting meta-properties such as *compactness* and *decidability*. Finite-valued PTS are also guaranteed to share those properties as soon as all the corresponding admissible translations are recursively defined. However, several logics known to be uncharacterizable by finite-valued truth-tables can be adequately characterized by finite-valued N-truth-tables, and several logics that have no finite-valued N-truth-tabular characterization can be characterized by finite-valued PTS.

The present contribution will examine and illustrate the multiple relations between the three above mentioned alternative styles of semantics.

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